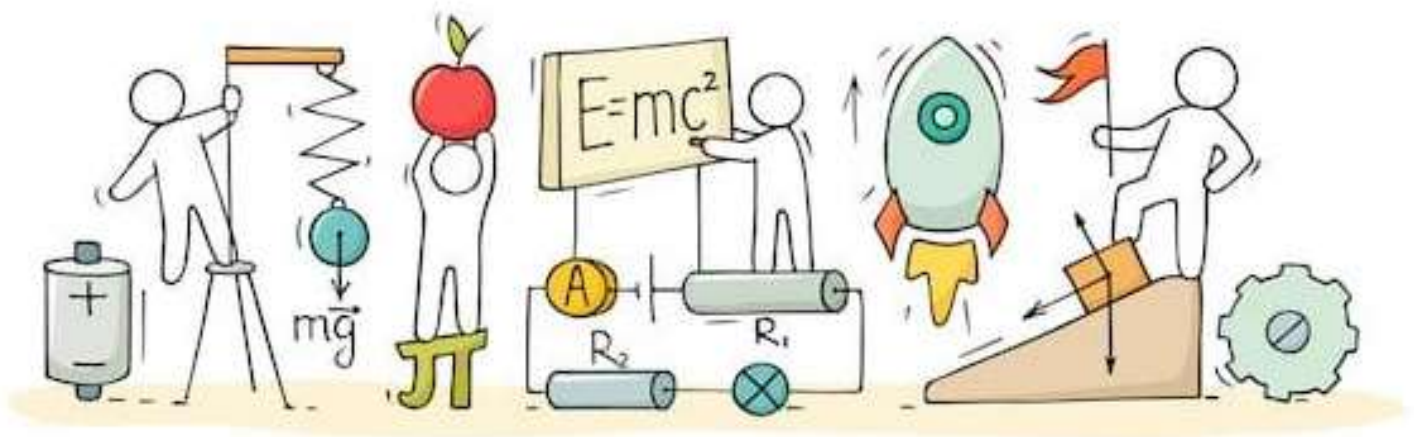


PHYSICS

Chapter 3: Motion in a Straight Line



Motion in a Straight Line

If an object changes its position with respect to its surroundings with time, then it is called in motion. It is a change in the position of an object over time. Motion in a straight line is nothing but linear motion. As the name suggests, it's in a particular straight line, thus it can be said that it uses only one dimension.

There are two branches in physics that examine the motion of an object.

- **Kinematics:** It describes the motion of objects, without looking at the cause of the motion.
- **Dynamics:** It relates the motion of objects to the forces which cause them.

Position, Distance, Displacement:

Position: Position of an object is always expressed with respect to some reference point which we generally account to as origin. To express the change in position, we consider two physical quantities.

Distance: It refers to the actual path traversed by the object during the course of motion.

Displacement: It refers to the difference between the final and initial positions of the object during the course of motion.

Distance	Displacement
It refers to the actual path traversed by the object during the course of motion.	It refers to the difference between the initial and the final positions $\Delta x = x_2 - x_1$, where, x_2 and x_1 are final and initial position respectively.
It is a scalar quantity.	It is a vector quantity.
The distance covered by an object during the course of motion can never be negative or zero. It is always positive.	The displacement of an object can be positive, negative or zero during the course of motion.
The distance travelled is either equal to or greater than displacement and is never less than magnitude of displacement.	The magnitude of displacement is less than or equal to the distance travelled during the course of motion.
The distance is dependent upon the path travelled by the object.	The magnitude of displacement is not dependent on the path taken by an object during the course of motion.

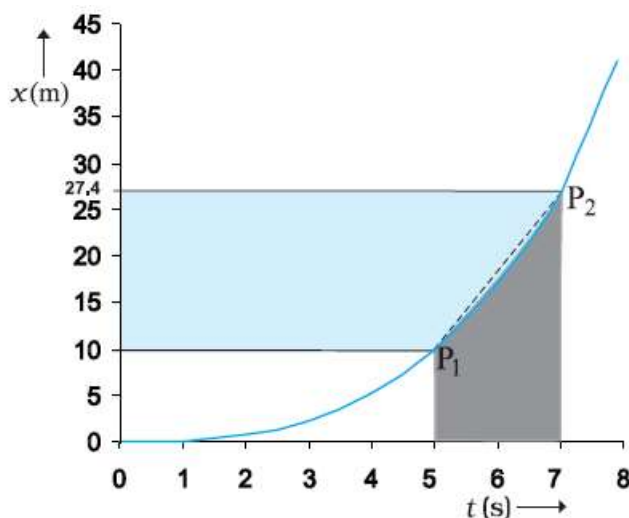
Average Velocity and Average Speed

Average velocity is defined as the change in position or displacement (Δx) divided by the time intervals (Δt), in which the displacement occurs:

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

where x_2 and x_1 are the positions of the object at time t_2 and t_1 , respectively. Here the bar over

the symbol for velocity is a standard notation used to indicate an average quantity. The SI unit for velocity is m/s or m s^{-1} , although km h^{-1} is used in many everyday applications.



The average velocity is the slope of line P1 P2

The portion of the x-t graph between $t = 0$ s and $t = 8$ s is blown up and shown in Fig. As seen from the plot, the average velocity of the car between time $t = 5$ s and $t = 7$ s :

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(27.4 - 10.0)\text{m}}{(7 - 5)\text{s}} = 8.7 \text{ m s}^{-1}$$

Average speed is defined as the total path length travelled divided by the total time interval during which the motion has taken place:

$$\text{Average speed} = \frac{\text{Total path length}}{\text{Total time interval}}$$

Average speed has obviously the same unit (m s^{-1}) as that of velocity. But it does not tell us in what direction an object is moving. Thus, it is always positive (in contrast to the average velocity which can be positive or negative). If the

motion of an object is along a straight line and in the same direction, the magnitude of displacement is equal to the total path length. In that case, the magnitude of average velocity is equal to the average speed.

However, this is not always the case. The average velocity gives an idea on how fast an object has been moving over a given interval but does not give an idea on how fast it moves at different instants of time during that interval.

Difference Between Speed and Velocity:

Speed	Velocity
It refers to the total path length travelled divided by the total time interval during which the motion has taken place.	It refers to the change in position or displacement divided by the time intervals, in which this displacement occurs.
It is a scalar quantity.	It is a vector quantity.
It is always positive during the course of the	It may be positive, negative or zero during the

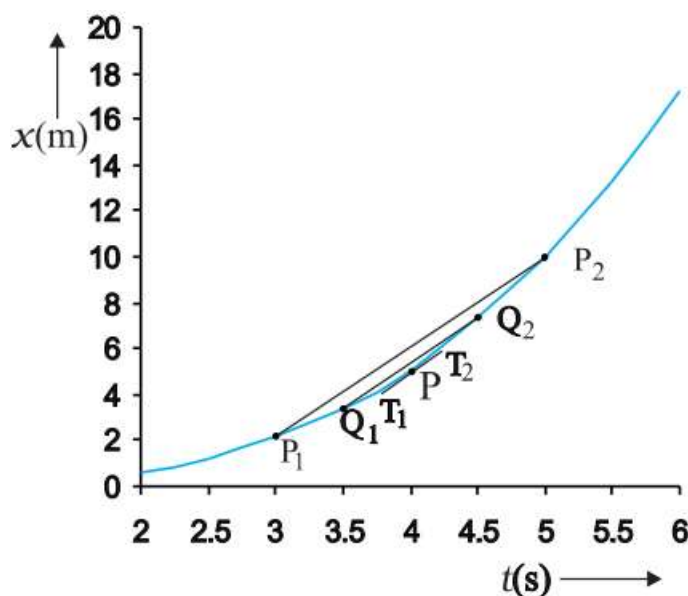
motion.	course of the motion.
It is greater than or equal to the magnitude of velocity.	It is less than or equal to the speed.

Instantaneous Velocity and Instantaneous Speed:

The velocity at an instant is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small. In other words,

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

where the symbol $\lim_{\Delta t \rightarrow 0}$ stands for the operation of taking limit as $\Delta t \rightarrow 0$ of the quantity on its right. In the language of calculus, the quantity on the right hand side is the differential coefficient of x with respect to t and is denoted by $\frac{dx}{dt}$. It is the rate of change of position with respect to time, at that instant.



Determining velocity from position-time graph. Velocity at $t = 4$ s is the slope of the tangent to the graph at that instant.

Acceleration

The **average acceleration** \bar{a} over a time interval is defined as the change of velocity divided by the time interval:

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

where v_2 and v_1 are the instantaneous velocities or simply velocities at time t_2 and t_1 . It is the average change of velocity per unit time. The SI unit of acceleration is m s^{-2} .

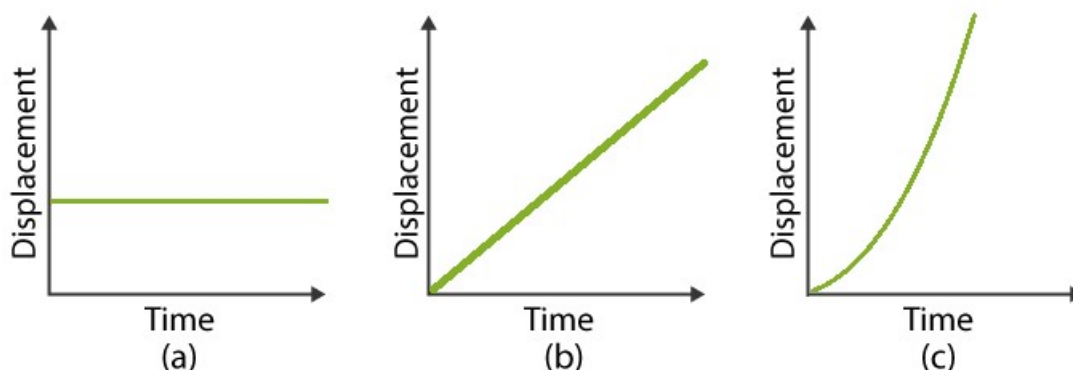
Instantaneous Acceleration: Mathematically, instantaneous acceleration can be expressed in the same way as the instantaneous velocity as follows:

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

The acceleration of an object at a particular time is the slope of the velocity-time graph at that instant of time. For uniform motion, acceleration is zero and the x-t graph is a straight line inclined to the time axis and the v-t graph is a straight line parallel to the time axis. For motion with uniform acceleration, x-t graph is a parabola while the v-t graph is a straight line inclined to the time axis.

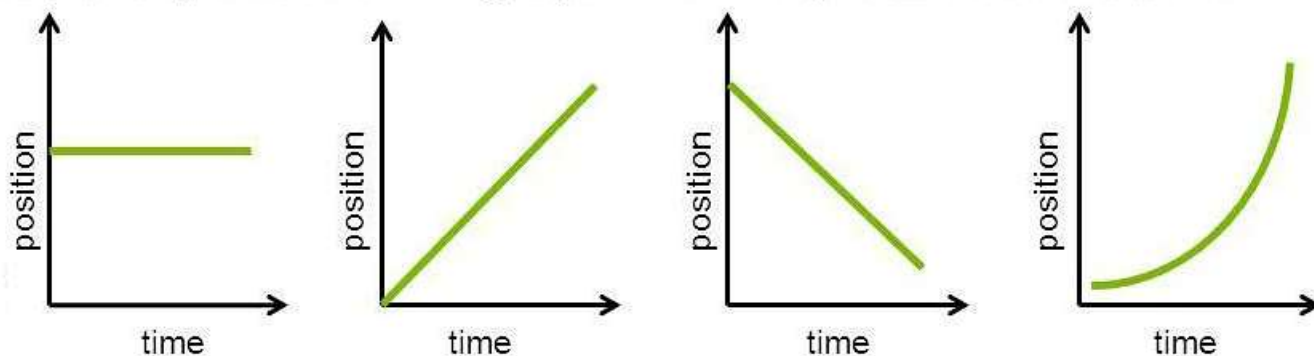
Different Graphs of Motion

Displacement - Time Graph



Velocity - Time Graph

Slope of position-time graph = velocity over that interval of time



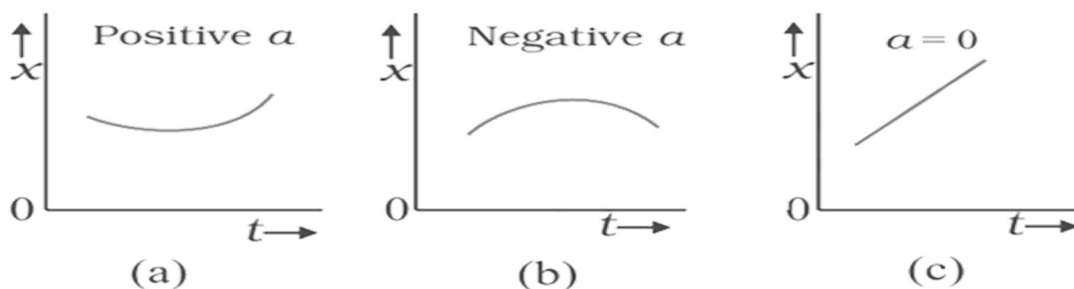
Slope is zero
 \therefore velocity is zero
 (object at rest)

Slope is positive
 \therefore velocity is
 constant, positive

Slope is negative
 \therefore velocity is
 constant, negative

Slope is curve
 \therefore velocity is **not**
 constant
 (object accelerating)

Acceleration - Time Graph



Position-time graph for motion with (a) positive acceleration; (b) negative acceleration, and (c) zero acceleration

Uniform motion:

If a body is said to be in uniform motion, the body completes equal distances in equal intervals of time.

Here, velocity is constant during the course of motion. Also, acceleration is zero during the course of motion.

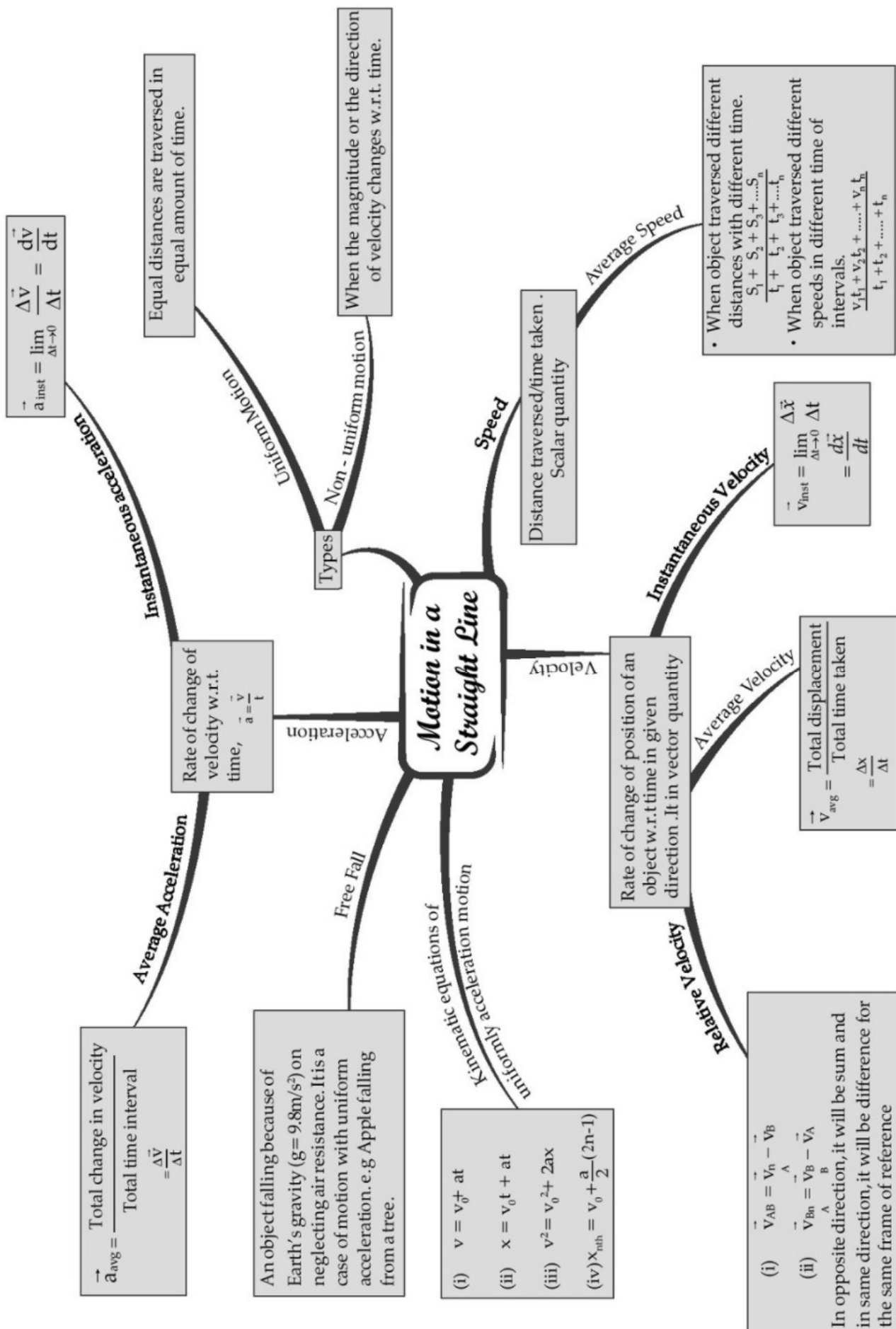
Non-Uniform motion:

If a body undergoes non-uniform motion, the body is said to be in uniformly accelerated motion. Here, the magnitude of velocity increases or decreases with the passage of time. Also, acceleration would not be zero as it undergoes accelerated motion.

Top Formulae

Displacement	$\Delta x = x_2 - x_1$
Average velocity	$\bar{v} = \frac{\text{Displacement}}{\text{time interval}} = \frac{\Delta x}{\Delta t}$
Instantaneous velocity	$v = \lim_{\Delta t \rightarrow 0} \bar{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$
Average acceleration	$\bar{a} = \frac{\Delta v}{\Delta t}$
Instantaneous acceleration	$a = \lim_{\Delta t \rightarrow 0} \bar{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$
Kinematic equations of motion	$v = v_0 + at$ $x = v_0 t + \frac{1}{2} at^2$ $v^2 = v_0^2 + 2ax$

CHAPTER - 3 : MOTION IN A STRAIGHT LINE



Important Questions

Multiple Choice questions-

1. A boy starts from a point A, travels to a point B at a distance of 3 km from A and returns to A. If he takes two hours to do so, his speed is
 - (a) 3 km/h
 - (b) zero
 - (c) 2 km/h
 - (d) 1.5 km/h
2. A body starts from rest and travels with uniform acceleration a to make a displacement of 6 m. If its velocity after making the displacement is 6 m/s, then its uniform acceleration a is
 - (a) 6 m/s^2
 - (b) 2 m/s^2
 - (c) 3 m/s^2
 - (d) 4 m/s^2
3. Which one of the following is the unit of acceleration?
 - (a) m/s
 - (b) m/s^2
 - (c) km/hr
 - (d) cm/s
4. The dimensional formula for speed is
 - (a) T^{-1}
 - (b) LT^{-1}
 - (c) $L^{-1}T^{-1}$
 - (d) $L^{-1}T$
5. A body starts from rest and travels for t second with uniform acceleration of 2 m/s^2 . If the displacement made by it is 16 m, the time of travel t is
 - (a) 4 s
 - (b) 3 s
 - (c) 6 s
 - (d) 8 s

6. The dimensional formula for acceleration is

- (a) $[LT^2]$
- (b) $[LT^{-2}]$
- (c) $[L^2T]$
- (d) $[L^2T^2]$

7. A body starts from rest and travels for five seconds to make a displacement of 25 m. if it has travelled the distance with uniform acceleration a then a is

- (a) 3 m/s^2
- (b) 4 m/s^2
- (c) 2 m/s^2
- (d) 1 m/s^2

8. A 180 meter long train is moving due north at a speed of 25 m/s. A small bird is flying due south, a little above the train, with a speed of 5 m/s. The time taken by the bird to cross the train is

- (a) 10 s
- (b) 12 s
- (c) 9 s
- (d) 6 s

9. The dimensional formula for velocity is

- (a) $[LT]$
- (b) $[LT^{-1}]$
- (c) $[L^2T]$
- (d) $[L^{-1}T]$

10. A body starts from rest and travels with an acceleration of 2 m/s^2 . After t seconds its velocity is 10 m/s. Then t is

- (a) 10 s
- (b) 5 s
- (c) 20 s
- (d) 6 s

Very Short:

1. Can a moving body have relative velocity zero with respect to another body? Give an example.

2. Can there be motion in two dimensions with acceleration in only one dimension?
3. Is it true that a body is always at rest in a frame that is fixed to the body itself?
4. Tell under what condition a body moving with uniform velocity can be in equilibrium?
5. What does the speedometer records: the average speed or the instantaneous speed?
6. Can an object be accelerated without speeding up or slowing down? Give examples,
7. Is it possible to have the rate of change of velocity constant while the velocity itself changes both in magnitude and direction? Give an example.
8. Which motion is exactly represented by $\Delta s = v\Delta t$?
9. In which frame of reference is the body always at rest?
10. What is common between the two graphs shown in figs, (a) and (b)?

Short Questions:

1. Prove that the average velocity of a particle over an interval of time is either smaller than or equal to the average speed of the particle over the same interval.
2. Two trains each of the length 109 m and 91 m are moving in opposite directions with velocities 34 km h^{-1} and 38 km h^{-1} respectively. At what time the two trains will completely cross each other?
3. Ambala is at a distance of 200 km from Delhi. Ram sets out from Ambala at a speed of 60 km h^{-1} and Sham set out at the same time from Delhi at a speed of 40 km h^{-1} . When will they meet?
4. A car travelling at a speed of 60 km h^{-1} on a straight road is ahead of a scooter travelling at a speed of 40 km h^{-1} . How would the relative velocity be altered if the scooter is ahead of the car?
5. Draw the position-time graphs for two objects initially occupying different positions but having zero relative velocity.
6. A ball is thrown vertically upward with a velocity of 20 ms^{-1} . It takes 4 seconds to return to its original position. Draw a velocity-time graph for the motion of the ball and answer the following questions:
At which point P, Q, R, the stone has:
(a) reached its maximum height.
(b) stopped moving?
7. "It is the velocity and not the acceleration which decides the direction of motion of a body." Justify this statement with the help of a suitable example.

8. Two buses A and B starting from the same point move in a mutually perpendicular direction with speeds $u_A \text{ km h}^{-1}$ and $u_B \text{ km h}^{-1}$ respectively. Calculate the relative velocity of A w.r.t B.

Long Questions:

1. Define the following terms:
 - (a) speed
 - (b) uniform speed
 - (c) variable speed
 - (d) average speed
 - (e) instantaneous speed
 - (f) velocity
 - (g) uniform velocity
 - (h) variable velocity
 - (i) uniform motion
 - (j) average velocity in uniform
 - (k) relative velocity motion
 - (l) instantaneous velocity
 - (m) acceleration
 - (n) retardation
 - (o) variable acceleration
 - (p) average acceleration
 - (q) uniform acceleration
 - (r) instantaneous acceleration.
2. Explain the importance of the position-time graph.
3. Derive relations:
 - (i) $v = u + at$
 - (ii) $v^2 - u^2 = 2as$
 - (iii) $s = ut + \frac{1}{2}at^2$.

Assertion Reason Questions:

1. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

- (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
- (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
- (c) Assertion is correct, reason is incorrect
- (d) Assertion is incorrect, reason is correct.

Assertion: A body may be accelerated even when it is moving uniformly.

Reason: When direction of motion of the body is changing, the body must have acceleration.

2. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

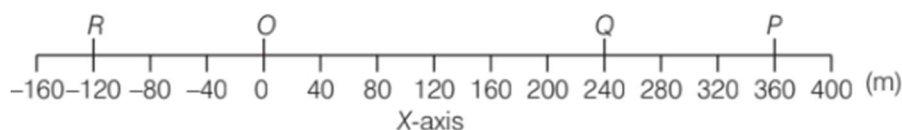
- (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
- (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
- (c) Assertion is correct, reason is incorrect
- (d) Assertion is incorrect, reason is correct.

Assertion: Displacement of a body may be zero when distance travelled by it is not zero.

Reason: The displacement is the longest distance between initial and final position.

Case Study Questions:

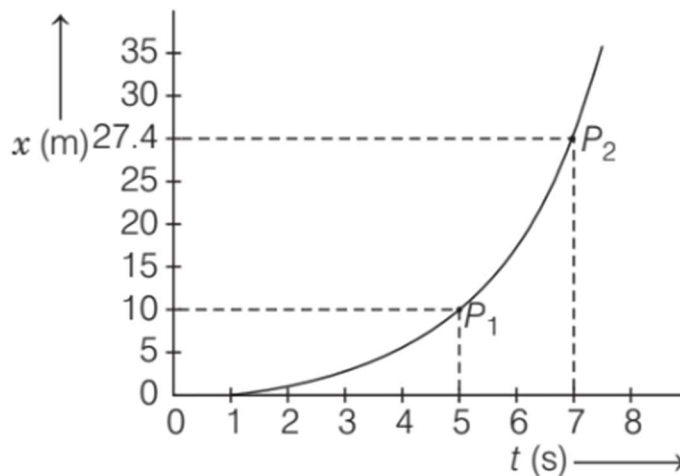
1. If the position of an object is continuously changing w.r.t. its surrounding, then it is said to be in the state of motion. Thus, motion can be defined as a change in position of an object with time. It is common to everything in the universe. In the given figure, let P, Q and R represent the position of a car at different instants of time.



- i. With reference to the given figure, the position coordinates of points P and R are
 - (a) $P = (+360, 0, 0)$; $R = (-120, 0, 0)$
 - (b) $P = (-360, 0, 0)$; $R = (+120, 0, 0)$
 - (c) $P = (0, +360, 0)$; $R = (-120, 0, 0)$
 - (d) $P = (0, 0 + 360)$ $R = (0, 0, -120)$
- ii. Displacement of an object can be
 - (a) Positive

- (b) Negative
 - (c) Zero
 - (d) All of the above
- iii. The displacement of a car in moving from O to P and its displacement in moving from P to Q are
- (a) + 360m and -120m
 - (b) -120m and + 360m
 - (c) + 360m and + 120m
 - (d) + 360m and - 600m
- iv. If the car goes from O to P and returns back to O, the displacement of the journey is
- (a) Zero
 - (b) 720m
 - (c) 420m
 - (d) 340m
- v. The path length of journey from O to P and back to O is
- (a) 0 m
 - (b) 720m
 - (c) 360m
 - (d) 480m
2. When an object is in motion, its position changes with time. So, the quantity that describes how fast is the position changing w.r.t. time and in what direction is given by average velocity. It is defined as the change in position or displacement (Δx) divided by the time interval (Δt) in which that displacement occurs. However, the quantity used to describe the rate of motion over the actual path, is average speed. It defined as the total distance travelled by the object divided by the total time taken.
- i. A 250m long train is moving with a uniform velocity of 4.5 kmh^{-1} The time taken by the train to cross a bridge of length 750m is
 - (a) 56 s
 - (b) 68 s
 - (c) 80 s
 - (d) 92 s
 - ii. A truck requires 3 hr to complete a journey of 150 km. What is average speed?

- (a) 50 km/h
(b) 25 km/h
(c) 15 km/h
(d) 10 km/h
- iii. Average speed of a car between points A and B is 20 m/s, between B and C is 15 m/s and between C and D is 10 m/s. What is the average speed between A and D, if the time taken in the? mentioned sections is 20s, 10s and 5s, respectively?
- (a) 17.14 m/s
(b) 15 m/s
(c) 10 m/s
(d) 45 m/s
- iv. A cyclist is moving on a circular track of radius 40 m completes half a revolution in 40 s. Its average velocity is
- (a) zero
(b) 2 ms^{-1}
(c) $4\pi \text{ ms}^{-1}$
(d) $8\pi \text{ ms}^{-1}$
- v. In the following graph, average velocity is geometrically represented by



- (a) Length of the line $P_1 P_2$
(b) Slope of the straight-line $P_1 P_2$
(c) Slope of the tangent to the curve at P_1
(d) Slope of the tangent to the curve at P_2

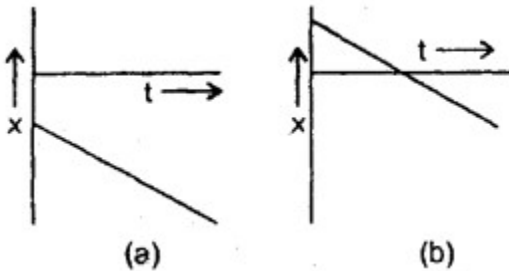
✓ **Answer Key:**

Multiple Choice Answers-

1. Answer: (a) 3 km/h
2. Answer: (c) 3 m/s^2
3. Answer: (b) m/s^2
4. Answer: (b) LT^{-1}
5. Answer: (b) 3 s
6. Answer: (b) $[\text{LT}^{-2}]$
7. Answer: (c) 2 m/s^2
8. Answer: (d) 6 s
9. Answer: (b) $[\text{LT}^{-1}]$
10. Answer: (b) 5 s

Very Short Answers:

1. Answer: Yes, two trains running on two parallel tracks with the same velocity in the same direction.
2. Answer: Yes, projectile motion.
3. Answer: Yes.
4. Answer: When the net force on the body is zero.
5. Answer: It records (or measures) the instantaneous speed.
6. Answer: Yes, circular motion.
7. Answer: Yes, in projectile motion.
8. Answer: It Represents motion with uniform velocity.
9. Answer: The body is always at rest in the frame attached to it i. e. inertial frame of reference.
10. Answer:



Both these graphs represent that the velocity is negative.

Short Questions Answers:

1. Answer: Average velocity is defined as the ratio of the total displacement to the total time. Average speed is defined as the ratio of the total distance to the total time. Since displacement is less than or equal to the distance, therefore the average velocity is less than or equal to the average speed.

2. Answer:

Let l_1, l_2 be the lengths of the two trains.

v_1, v_2 be their velocities respectively.

$$\therefore l_1 = 109\text{m}, l_2 = 91\text{ m}, v_1 = 34\text{kmh}^{-1}, v_2 = 38\text{kmh}^{-1}.$$

As the trains are moving in opposite directions so relative velocity of the trains is given by

$$v_1 - (-v_2) = v_1 + v_2$$

$$= 34 + 38 = 72\text{ kmh}^{-1}$$

$$= 72 \times \frac{5}{18} = 20\text{ ms}^{-1}$$

Total distance to be covered by the two trains in crossing each other

$$= l_1 + l_2 = 109 + 91 = 200\text{ m}$$

If t be the time taken in crossing, then t can be calculated using the relation

$$x = vt$$

or

$$t = \frac{200}{20} = 10\text{s}$$

3. Answer:

$S = 200\text{ km}$. Let V_R and v_s be the speeds of Ram and Sham respectively moving in opposite directions.

$$\therefore v_R = 60\text{ kmh}^{-1}, v_s = 40\text{ kmh}^{-1}.$$

\therefore Relative velocity of Ram w.r.t. Sham is

$$V_{RS} = V_R - (-V_S)$$

$$= V_R + V_S$$

$$= 60 + 40 = 100\text{ kmh}^{-1}$$

If t = time after which they will meet, then

t = time taken in covering 200 km distance with VRS

$$\text{i.e. } t = \frac{200}{v_{RS}} = \frac{200\text{ km}}{100\text{kmh}^{-1}} = 2\text{h}.$$

\therefore Time after which they meet = 2h.

4. Answer:

v_c = speed of car = 60 kmh^{-1}

v_s = speed of scooter = 40 kmh^{-1}

v_{cs} = relative velocity of car w.r.t. scooter

$$= v_c - v_s$$

$$= 60 - 40$$

$$= 20 \text{ kmh}^{-1}$$

Similarly, v_{sc} = relative velocity of scooter w.r.t. car

$$= v_s - v_c$$

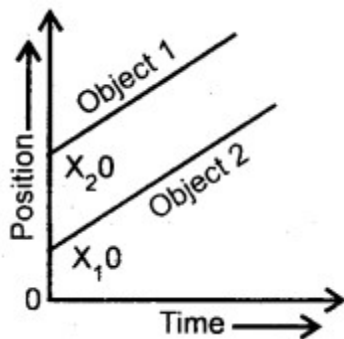
$$= 40 - 60$$

$$= -20 \text{ kmh}^{-1}$$

Thus, we conclude that the magnitude of the relative velocity is the same in both cases but the direction of relative velocity is reversed if the scooter is ahead of the car.

5. Answer:

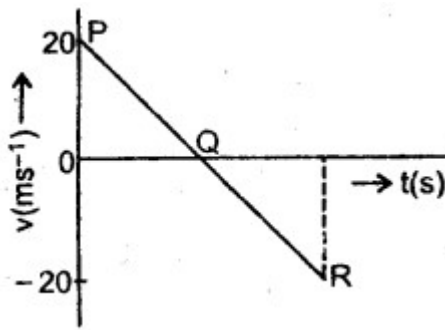
The positive T time graphs for two objects initially occupying different positions but having zero relative velocity are parallel to each other as shown in Fig.



6. Answer:

- Let P represent the initial position at the time when the ball is thrown vertically upward.
- Q represents the highest point reached by the ball.
- R represents the original position of the ball after 4 seconds.

Thus, the velocity-time graph for the motion of the ball is as shown in Fig.



(a) We know that at the highest point, the velocity of the object is zero. So, stone will reach its maximum height corresponding to point Q.

(b) The stone has stopped moving at point Q because at Q, $v = 0$.

7. Answer:

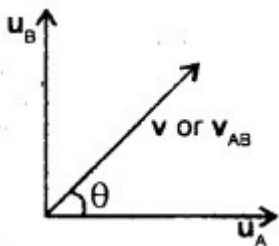
The direction of velocity is always in the direction of motion of the body whereas the direction of acceleration may or may not be in the direction of motion of the body. Thus we conclude that it is the velocity that decides the direction of motion of the body.

Example: When a ball is thrown vertically upwards, the direction of motion of the ball and velocity is the same i.e. vertically upwards. On the other hand, the acceleration due to gravity on the ball acts vertically downwards i.e. opposite to the direction of motion of the ball.

8. Answer:

Since u_A and u_B are in mutually perpendicular directions, they will cover u_A and u_B km in one hour respectively. Thus if v km be the separation between them in one hour,

$$\text{Then } v = \sqrt{u_A^2 + u_B^2}$$



Thus, if v_{AB} be the relative speed of A w.r.t. B, then

$$v_{AB} = \sqrt{u_A^2 + u_B^2} \text{ kmh} \dots (1)$$

If θ be the direction of v_{AB} w.r.t. u_A , Then

$$\tan \theta = \frac{u_B}{v_{AB}} = \frac{u_B}{\sqrt{u_A^2 + u_B^2}} \dots (2)$$

Thus, equations (1) and (2) give the magnitude and direction of relative velocity of A w.r.t. B.

Long Questions Answers:

1. Answer:

(a) Speed: It is defined as the time rate of change of position i. e. distance of an object.

$$\text{i.e. Speed} = \frac{\text{Distance travelled by the object}}{\text{Time taken}}$$

(b) Uniform Speed: An object is said to be moving with uniform speed if it covers equal distances in equal small intervals of time.

(c) Variable Speed: An object is said to be moving with variable speed if it covers equal distances in unequal small intervals of time.

(d) Average Speed: It is used to measure the variable speed of an object.

It is defined as the ratio of the total distance travelled by the object to the total time taken.

$$\therefore v_{av} = \frac{\text{Total}}{\text{Total time taken}}$$

(e) Instantaneous Speed: It is defined as the speed of an object at a given instant of time. It is denoted by v_{ins} .

\therefore If Δs be the distance covered by an object in a small-time interval Δt s.t. $\Delta t \rightarrow 0$,

Then

$$v_{ins} = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Thus in the case of the uniform motion of an object, the instantaneous speed is equal to its uniform speed.

(f) Velocity: It is defined as the time rate of change of displacement of an object.

(g) Uniform Velocity: An object is said to be moving with uniform velocity if it undergoes equal displacements in equal intervals of time however small these intervals may be.

(h) Variable Velocity: An object is said to be moving with variable velocity if either its magnitude (i.e. speed) or its direction or both change with time.

(i) Uniform Motion: An object is said to be in uniform motion if it undergoes equal displacements in equal intervals of time which may be very small.

(j) Average Velocity in Uniform Motion: The velocity of an object in uniform motion may be defined as the ratio of the displacement of the object to the total time interval for which the motion takes place.

$$\text{i.e. } v = \frac{x_2 - x_1}{t_2 - t_1}$$

(k) Relative Velocity: The relative velocity of a moving object with respect to another object is defined as the rate of change of relative position of one object w.r.t. another object.

Or

It is the velocity with which one object moves with respect to another object.

(l) The instantaneous velocity of an object: It is defined as the velocity of an object at any instant of time or any point on its path.

Or

It is defined as the limiting value of the average velocity of the object as $\Delta t \rightarrow 0$.

$$\begin{aligned} \text{i.e.} \quad v_{\text{ins}} &= \lim_{\Delta t \rightarrow 0} v_{\text{av}} \\ &= \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \end{aligned}$$

(m) Acceleration: It is defined as the time rate of change of velocity of an object. It is a vector quantity.

(n) Retardation: It is defined as the negative acceleration produced in the object.

(o) Variable Acceleration: An object is said to be moving with variable acceleration if its velocity changes by unequal magnitudes in equal intervals of time.

(p) Average Acceleration: It is defined as the ratio of change in velocity in a given time interval to the total time taken.

(q) Uniform Acceleration: An object is said to be moving with uniform acceleration if it undergoes equal changes in velocity in equal intervals of time.

(r) Instantaneous Acceleration: It is defined as the acceleration of an object at a particular instant of time or at a particular point on its path.

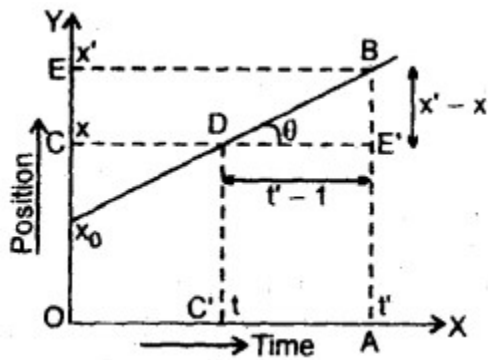
Or

It may be defined as the limiting value of the average acceleration in a small time interval around that instant when the time-interval tends to zero.

$$\begin{aligned} \text{i.e.} \quad a_{\text{ins}} &= \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \\ &= \frac{d}{dt}(v) = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \end{aligned}$$

2. Answer:

1. The importance of a position-time graph is that its slope gives the velocity of the object in uniform motion.



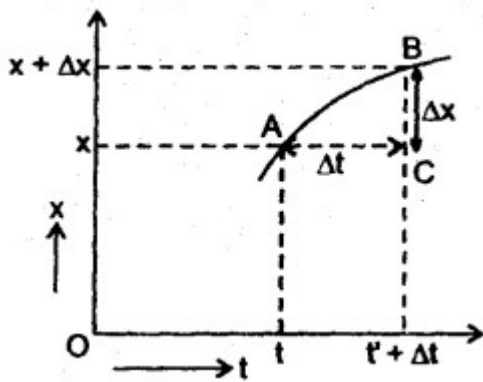
Let us consider the position-time graph of an object moving with uniform velocity represented by the line DB making angle θ with the time axis. Let the coordinates of D and B be (x, t) and (x', t') respectively. Let BA and DC' be perpendiculars drawn from B and D respectively on the time axis and BE and DC be perpendiculars on the y-axis from B and D respectively.

Now $BE' = CE = x' - x$

and $C'A = t' - t = DE'$

Then velocity $= \frac{x' - x}{t' - t} = \frac{BE'}{DE'} = \tan\theta$

So, velocity $v = \text{slope of position-time graph.}$



2. The position-time graph for a stationary object is a straight line parallel to the time axis. Here the slope of the curve is zero, which means the object is stationary as $v = 0$.

3. In the case of variable velocity the position time curve is not a straight line. In this case, the slope of the curve gives the average velocity

$$v_{\text{average}} = \frac{(x + \Delta x) - x}{(t + \Delta t) - t} = \frac{\Delta x}{\Delta t}$$

= slope of chord AB when $\Delta t \rightarrow 0$,

then the slope of curve gives the instantaneous velocity.

Instantaneous velocity =

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

Thus, the position-time graph gives information about velocity.

3. Answer:

$$v = u + at:$$

Derivation: By def. of acceleration, we know that

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

$$\text{or } v_2 - v_1 = a(t_2 - t_1)$$

$$\text{or } v_2 = v_1 + a(t_2 - t_1) \quad \dots (1)$$

where v_1 and v_2 are the velocities of an object at times t_1 and t_2 respectively.

If $v_1 = u$ (initial velocity of the object) at $t_1 = 0$

$v_2 = v$ (final velocity of the object) at $t_2 = t$

Then (1) reduces to $v = u + at$

Hence derived.

ii) $v_2 - u_2 = 2as$

Derivation: We know that acceleration is given by $a = \frac{v_2 - v_1}{t_2 - t_1}$, where v_1 and v_2 , t_1 and t_2 are as in (1).

$$\text{or } t_2 - t_1 = \frac{v_2 - v_1}{a} \quad \dots (1)$$

Also we know that

$$x_2 - x_1 = v_1(t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2 \quad \dots (2)$$

\therefore From (1) and (2), we get

$$\begin{aligned} x_2 - x_1 &= v_1 \frac{(v_2 - v_1)}{a} + \frac{1}{2}a \left(\frac{v_2 - v_1}{a} \right)^2 \\ &= \frac{v_1 v_2 - v_1^2}{a} + \frac{v_2^2 + v_1^2 - 2v_1 v_2}{2a} \\ &= \frac{2v_1 v_2 - 2v_1^2 + v_1^2 + v_2^2 - 2v_1 v_2}{2a} \\ &= \frac{v_2^2 - v_1^2}{2a} \end{aligned}$$

$$\text{or } v_2^2 - v_1^2 = 2a(x_2 - x_1) \quad \dots (3)$$

$$\text{Now if } \left. \begin{array}{l} v_1 = u \text{ at } t_1 = 0 \\ v_2 = v \text{ at } t_2 = t \end{array} \right\} \quad \dots (4)$$

$$x_2 - x_1 = s$$

Then from (3) and (4), we get

$$v^2 - u^2 = 2as \quad \dots (5)$$

$$s = ut + \frac{1}{2}at^2$$

$$(iii) s = ut + \frac{1}{2}at^2.$$

Derivation:

Let x_1, v_1 = position and velocity of the object at time t_1 .

x_2, v_2 = position and velocity of the object at time t_2 .

a = uniform acceleration of the object.

Also Let v_{av} = average velocity in $t_2 - t_1$ interval

\therefore By definition

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$\text{or } x_2 - x_1 = v_{av}(t_2 - t_1) \quad \dots (1)$$

Also we know that

$$v_{av} = \frac{v_1 + v_2}{2} \quad \dots (2)$$

\therefore From (1) and (2), we get

$$x_2 - x_1 = \frac{v_1 + v_2}{2}(t_2 - t_1) \quad \dots (3)$$

Also we know that

$$v_2 = v_1 + a(t_2 - t_1) \quad \dots (4)$$

\therefore From (3) and (4), we get

$$\begin{aligned} x_2 - x_1 &= \frac{1}{2}[v_1 + v_1 + a(t_2 - t_1)](t_2 - t_1) \\ &= v_1(t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2 \quad \dots (5) \end{aligned}$$

Now if $x_1 = x_0$ at $t_1 = 0$

$$\left. \begin{aligned} x_2 &= x \text{ at } t_2 = t \\ v_1 &= u \text{ at } t_1 = 0 \\ v_2 &= v \text{ at } t_2 = t \end{aligned} \right\} \dots (6)$$

\therefore From (5) and (6), we get

$$x - x_0 = ut + \frac{1}{2}at^2$$

if $x - x_0 = S$, then

$$s = ut + \frac{1}{2}at^2$$

Assertion Reason Answer:

1. (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

Explanation:

In uniform circular motion, there is acceleration of constant magnitude.

2. (c) Assertion is correct, reason is incorrect

Explanation:

The displacement is the shortest distance between initial and final position. When final position of a body coincides with its initial position, displacement is zero, but the distance travelled is not zero.

Case Study Answer:

1. i (a) $P = (+360, 0, 0)$; $R = (-120, 0, 0)$

Explanation:

The position coordinates of point $P = (+360, 0, 0)$ and point $R = (-120, 0, 0)$

- ii (d) All of the above

Explanation:

Displacement is a vector quantity, it can be positive, negative and zero.

- iii (a) $+360$ m and -120 m

Explanation:

Displacement, $\Delta x = x_2 - x_1$

For journey of car in moving from O to P ,

$$x_2 = + 360 \text{ m}$$

$$x_1 = 0$$

$$\Rightarrow \Delta x = x_2 - x_1 = 360 - 0 = + 360 \text{ m}$$

For journey, of car in moving from P to Q ,

$$x_2 = + 240 \text{ m}$$

$$x_1 = + 360 \text{ m}$$

$$\Rightarrow \Delta x = x_2 - x_1 = 240 - 360 = -120 \text{ m}$$

Here, -ve sign implies that the displacement is in -ve direction, i.e. towards left.

iv (a) zero

Explanation:

$$\text{Displacement, } \Delta x = x_2 - x_1 = 0 - 0 = 0$$

v (b) 720m

Explanation:

Path length of the journey

$$= OP + PO = + 360\text{m} + (+360) \text{ m} = 720\text{m}$$

2. i (c) 80 s

Explanation:

$$\text{Total time taken} = \frac{\text{The distance}}{\text{Speed}}$$

$$t = \frac{250 + 750}{45 \times \frac{5}{18}} = 80 \text{ s}$$

ii (a) 50 km/h

Explanation:

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{150}{3} = 50 \text{ km/h}$$

iii (a) 17.14 m/s

Explanation:

Total distance ($d = vt$)

$$= 20 \times 20 + 15 \times 10 + 10 \times 5 = 600 \text{ m}$$

Total time = $20 + 10 + 5 = 35 \text{ s}$

Therefore, average speed

$$= 600 / 35 = 17.14 \text{ m/s}$$

iv (b) 2 ms^{-1}

Explanation:

Given, $R = 40 \text{ m}$ and $t = 40 \text{ s}$

$$\begin{aligned} \text{Average velocity} &= \frac{\text{Displacement}}{\text{Time taken}} \\ &= \frac{2R}{t} = \frac{2 \times 40}{40} = 2 \text{ ms}^{-1} \end{aligned}$$

v (b) Slope of the straight line $P_1 P_2$

Explanation:

From the position-time graph, average velocity is geometrically represented by the slope of curve, i.e., slope of straight line $P_1 P_2$